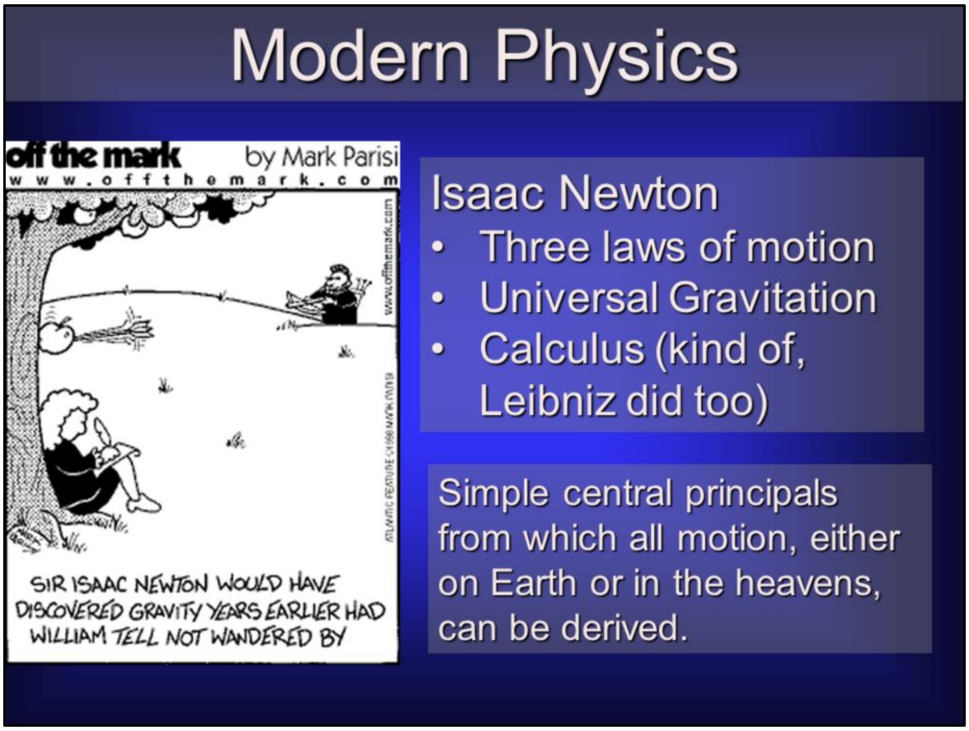


Gottfried Leibniz also invented calculus.

India had a long history of trigonometry as witnessed by the 8th century BCE treatise Sulba Sutras, or rules of the chord, where the sine, cosine, and tangent were conceived. Indian mathematicians gave a semi-rigorous method of differentiation of some trigonometric functions. In the Middle East, Alhazen derived a formula for the sum of fourth powers. He used the results to carry out what would now be called an integration, where the formulas for the sums of integral squares and fourth powers allowed him to calculate the volume of a paraboloid.[7] In the 14th century, Indian mathematician Madhava of Sangamagrama and the Kerala school of astronomy and mathematics stated components of calculus such as the Taylor series and infinite series approximations

In the 17th century, European mathematicians Isaac Barrow, René Descartes, Pierre de Fermat, Blaise Pascal, John Wallis and others discussed the idea of a derivative.

On the integral side, Bonaventura Cavalieri developed his method of indivisibles in the 1630s and 1640s. Evangelista Torricelli extended this work to other curves such as the cycloid, and then the formula was generalized to fractional and negative powers by Wallis in 1656. In a 1659 treatise, Fermat is credited with an ingenious trick for evaluating the integral of any power function directly

Importantly, Newton and Leibniz did not create the same Calculus and they did not conceive of modern Calculus.

Aristotle says:

The natural state of objects on the Earth is a state of rest.

Newton Says:

The natural state of motion of an object is to keep doing what it's doing.

<u>Galileo had already said this.</u>

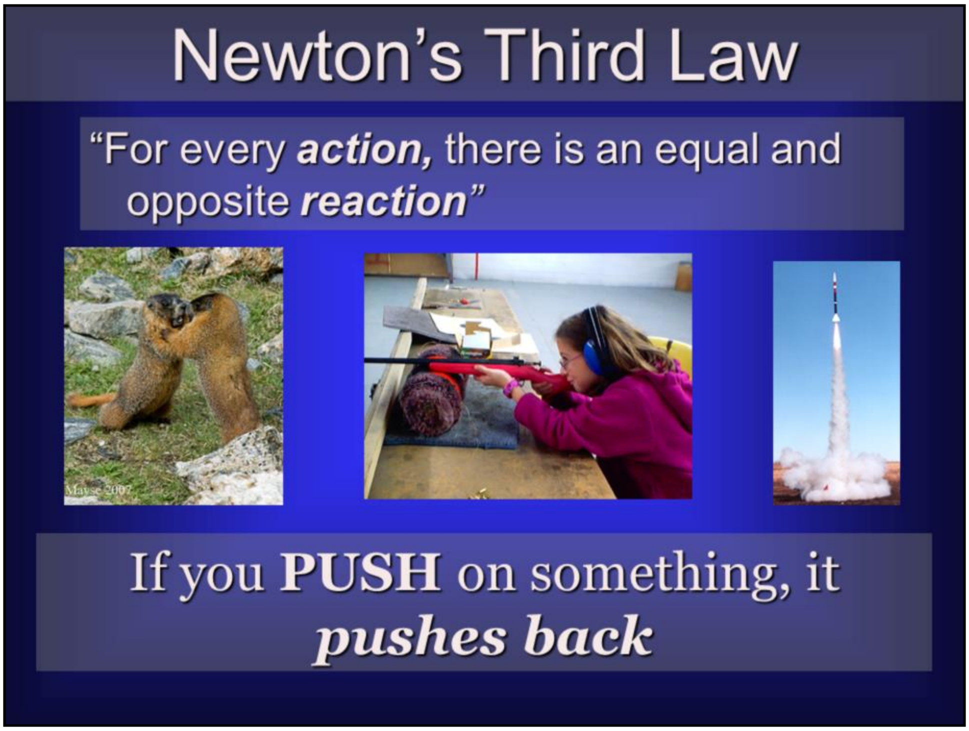
Aristotle divided the universe into "The Heavens" and "The Earth"

The "Heavens" were eternal and unchanging. The natural state of the heavens was perfect motion.

The Earth was temporary and ever changing.

The natural state of motion of Earth bound objects was rest.

Newton (and Galileo) said that the natural state of motion is to not change.



Forces come in pairs.

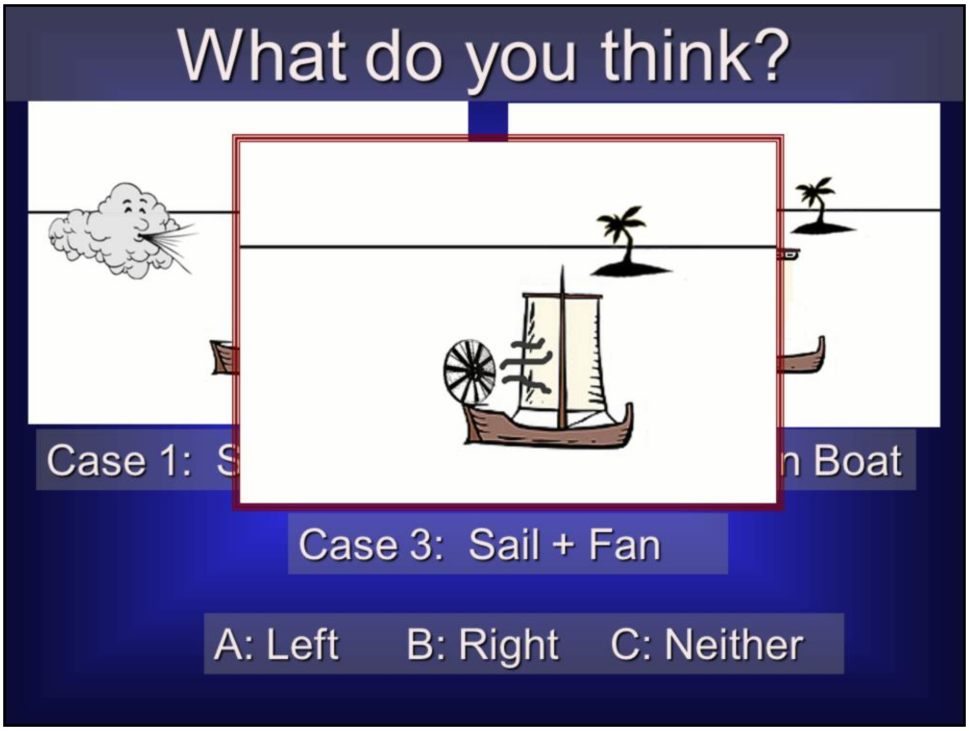
If you apply a force to something, it responds by pushing back.

As I push against a baseball to throw it, I can FEEL it pushing back.

This is how rockets work. We push mass out the back of the rocket and the opposing force pushes back, propelling the rocket forward.

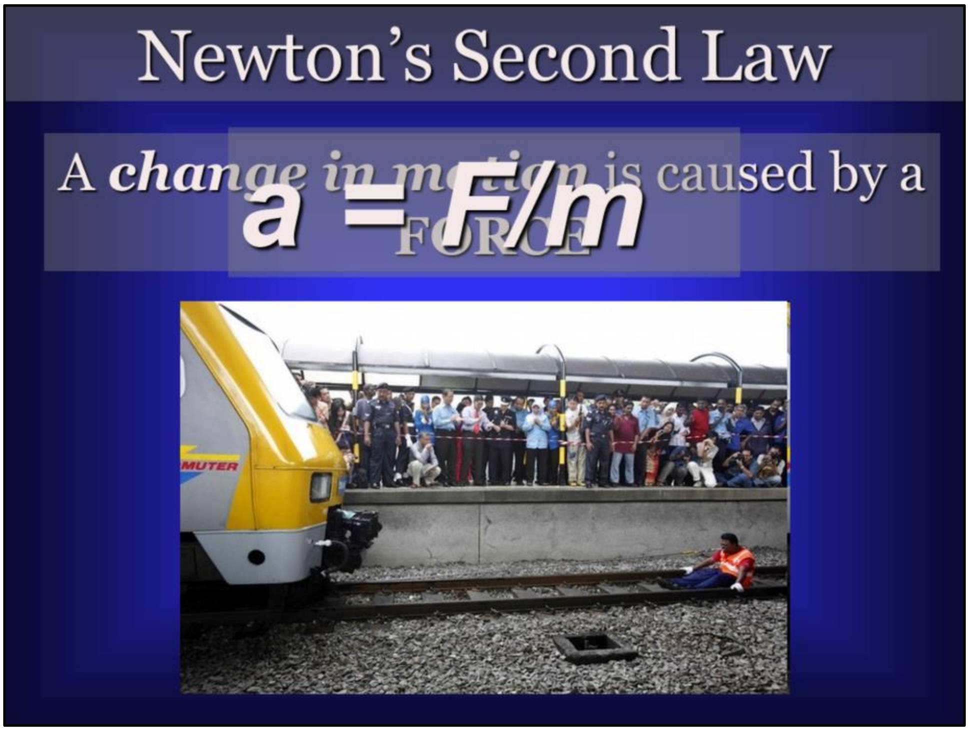
A few examples of force pairs:
Standing on the ground (ground pushes on me; I push on the ground)
Pushing on a wall (I push on wall; wall pushes on me)
Bug hitting a windshield
Throwing a baseball

The *effects* of those forces can be vastly different, but the *forces* are the same



Which direction will the boat move, and why?

Discuss Case 3 first, then vote. Revote after additional discussion if needed.

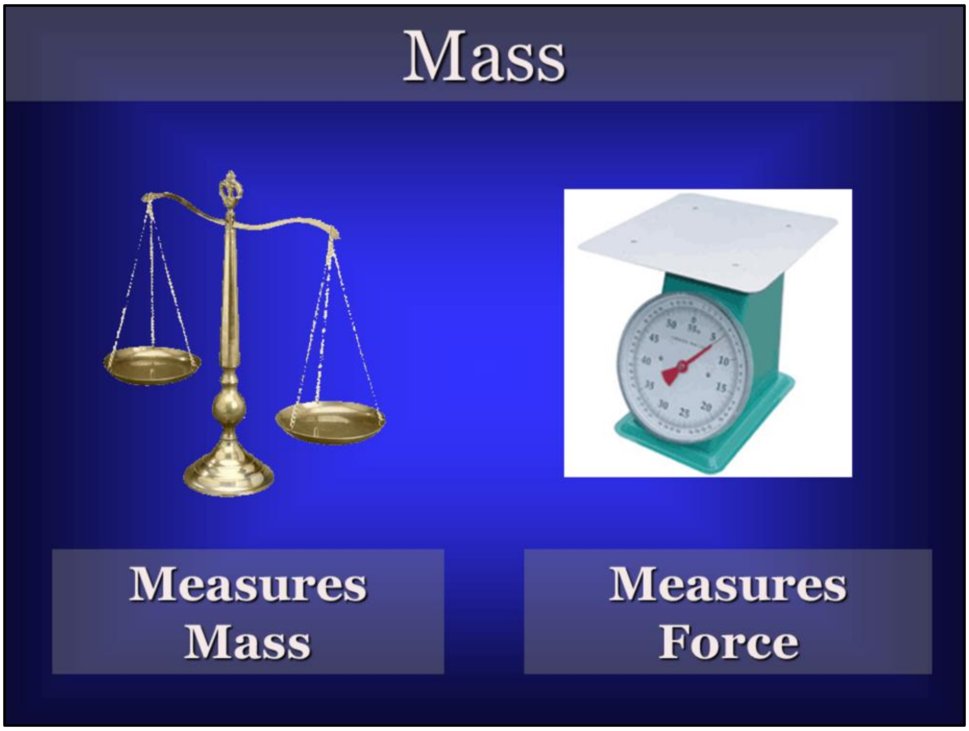


Galileo didn’t say this.

To get an object to change, a force must be applied.

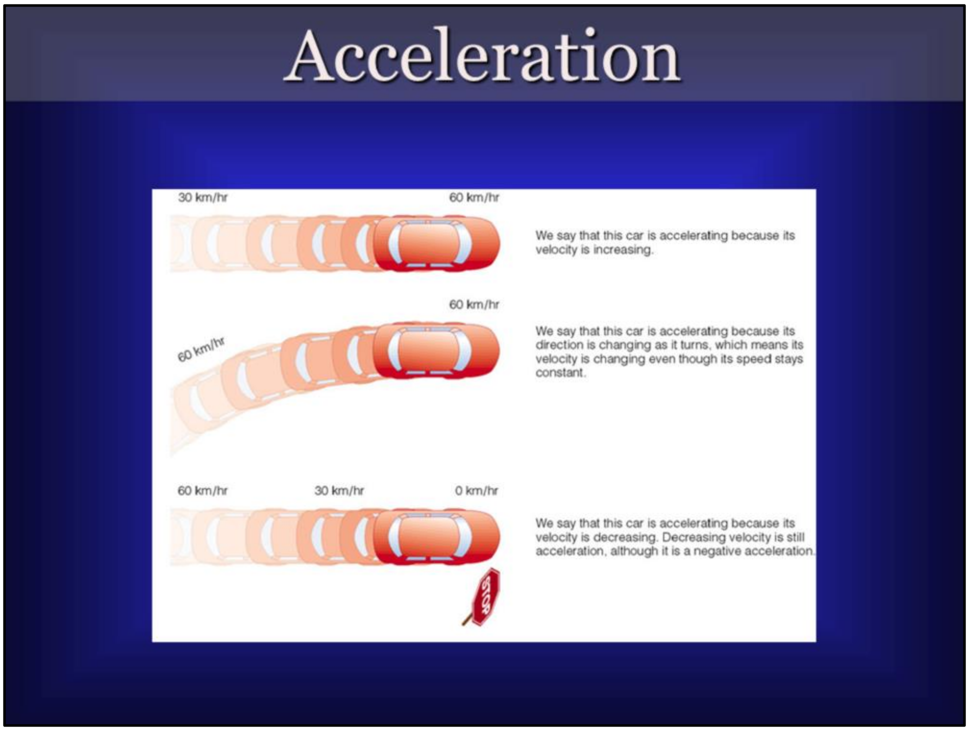
When we push the gas pedal in the car, the engine generates a force causing the car to move.

Why does the car stop if I stop pushing with the engine?



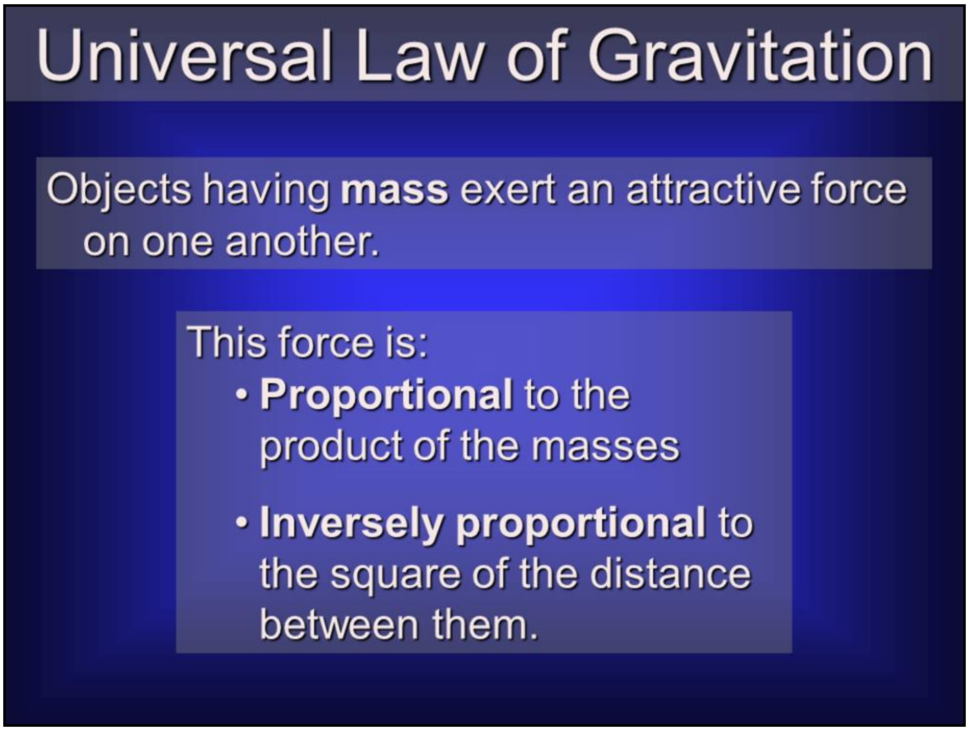
Balance scale:
Balance test object against a REFERENCE mass
Independent of gravitational force.
You’ll get the same answer on Mars as on Earth

Spring scale.
Measurement is dependent on gravitational force.
Measures how far a spring is stretched or compressed,
which is a measure of FORCE.
You’ll get DIFFERENT answers on Earth versus Mars



So… forces cause changes in **velocity**… or accelerations.

The acceleration is proportional to the force… and inversely proportional to the mass.



Phew! What a mouthful! What does it mean?
Give proportional examples: number of apple slices is proportional to the number of apples I start with. Doubling a recipe: I need to double each ingredient (usually). Making skirts – I need twice as much fabric to make twice as many skirts for myself.
Give inverse proportional examples: time needed to dig a hole is inversely proportional to the number of people digging. As temperature increases, sale of sweaters decreases. Etc.

Universal Law of Gravitation

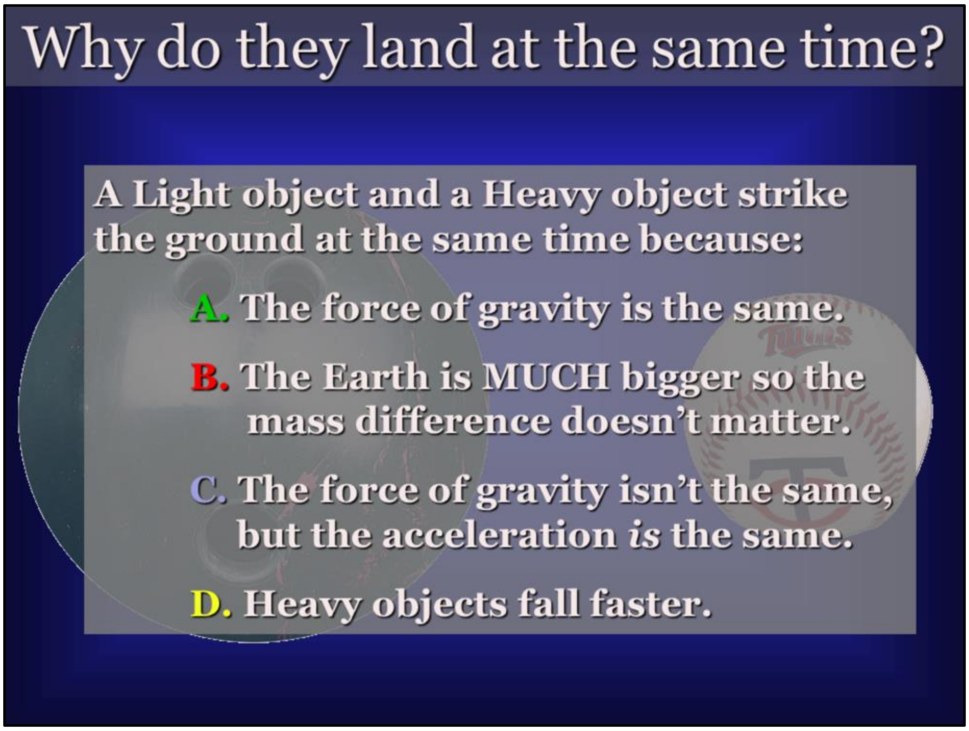
$$F = G\frac{M_1 M_2}{r^2}$$

Objects with **mass** exert an attractive force on one another.

But often on Earth we see: F = m g. The “m” is the mass of the object you are dropping.
What is “g”? $g = G\, M_{earth} / r_{earth}^2$

g is also the *acceleration* of whatever you’re dropping. Remember F = m a!

Exercise… Reason it out… not just a prediction.
I want the answer to talk about gravitation, mass, and acceleration.

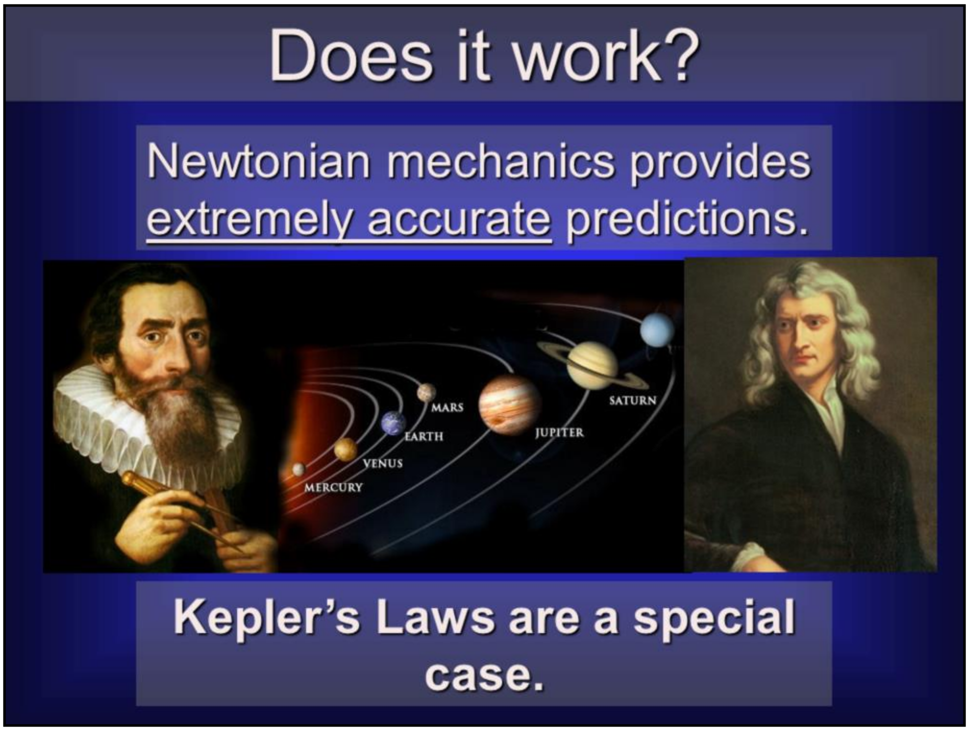


Fg, bowling ball = m bowling ball * g since the mass is larger than the baseball, the force is <u>larger</u>.

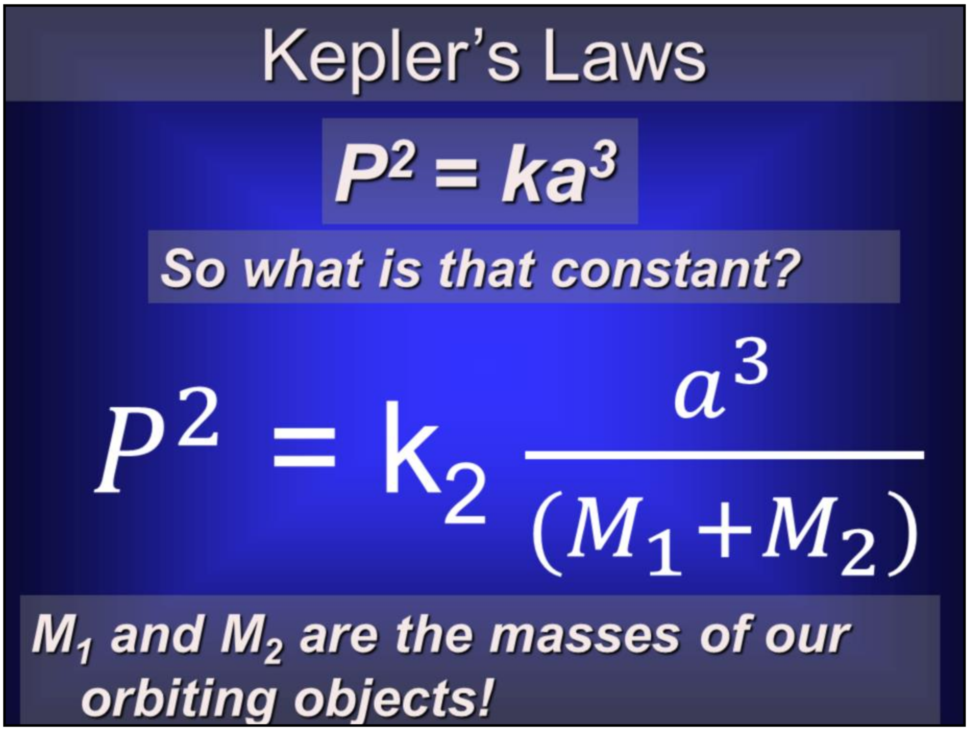
Is the earth much bigger? Yes. But twice the mass still is twice the force.

However the "g" is the same! That only depends on the mass of the Earth and the distance, which i

Heavy objects, it turns out, do NOT fall faster. Even if you want them to!



Extremely accurate, that is, until we get near something really REALLY massive, or start moving very VERY fast (~1/3 the speed of light). But that doesn’t happen every day.



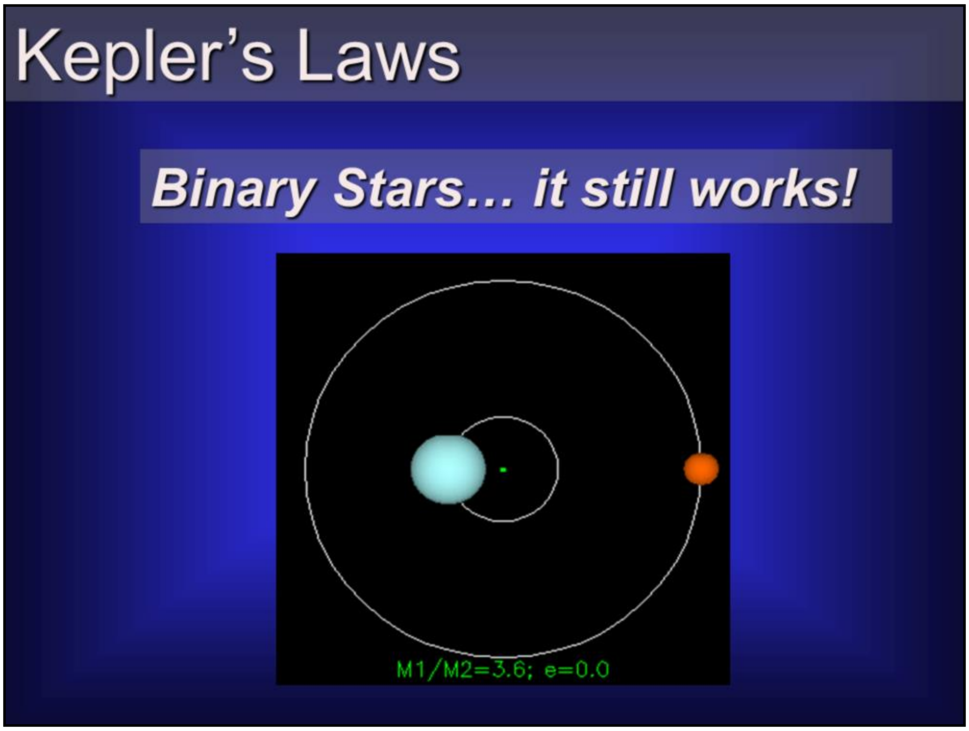
Our constant is just the mass of our two bodies orbiting each other! But why do we get the same thing for all the planets? Jupiter has a lot more mass than Earth, so why does this still work?
Even though Jupiter has a lot more mass than Earth, it is still MUCH MUCH smaller than the mass of the Sun! So, we can ignore the M2 in the equation because it's so much smaller than M1. M1+M2 is the total mass of the system.

NOTE: we are using **special units** (P in years, a in AU). If we do this, the mass comes out in Solar Masses (our sun is 1 solar mass). But what if we use real units?

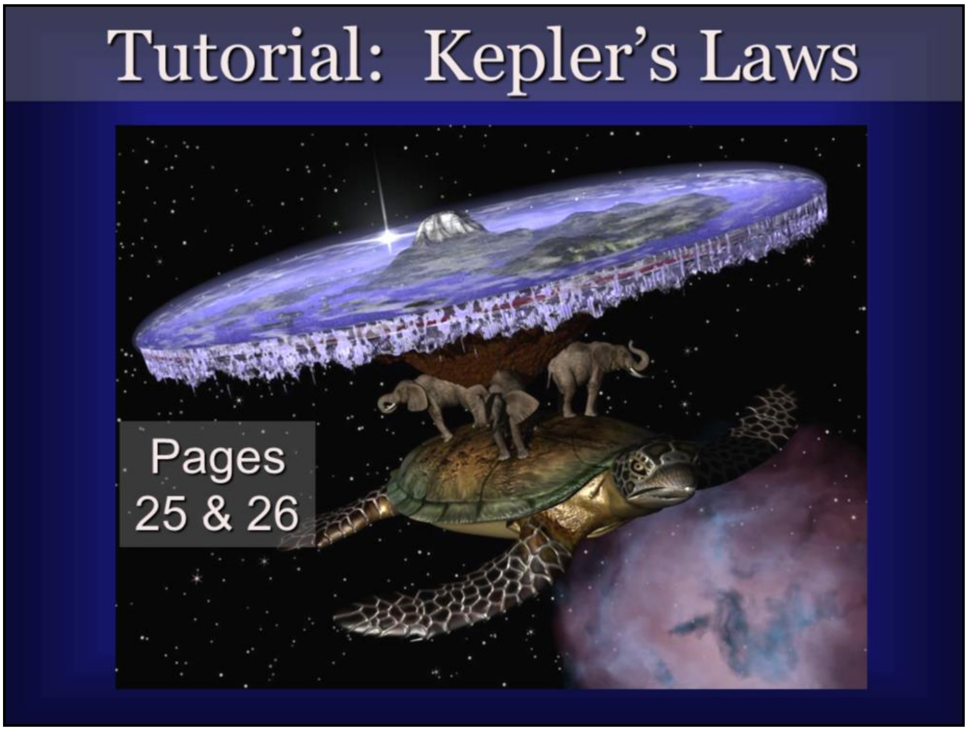
Kepler's Third Law

$$P^2 = \frac{G}{4\pi}\frac{a^3}{(M_1+M_2)}$$

Now we can use real units! G = 6.67 x 10^{-11} $m^3/kg/s^2$



Now I can't ignore the mass of the secondary object… it's too big!



Class discussion:
Page 21; Kepler's 2^{nd} Law tutorial
SKIP Page 22
Page 23: Kepler's 2^{nd} Law tutorial
Page 24: Kepler's 2^{nd} Law tutorial

Ask: Which one changes the most (percentage-wise)?

Tutorial: Kepler's 3^{rd} Law <u>pages 25 and 26 only</u>